# HEAT-TRANSFER CHARACTERISTICS OF PACKED BEDS WITH STAGNANT FLUIDS

# **KUNIHIKO OFUCHI and DAIZO KUNII**

Department of Chemical Engineering, University of Tokyo, Tokyo, Japan

#### *(Received* 21 December *1964)*

Abstract-Temperature distributions were measured in steady state in packed beds of solid particles with stagnant fluids, from which both the effective thermal conductivities and the apparent wall-film coefficients of heat transfer were obtained. Measurements were made for each of twelve kinds of particles with five fluids, i.e. water, air, carbon dioxide, helium and hydrogen.

Experimental data of the effective thermal conductivities showed good agreement with the theoretical equations proposed by the authors before.

The authors' theoretical equation for the apparent wall-film coefficient of heat transfer was modified here and applied to the analysis of the present experimental data.

# NOMENCLATURE

 $C_{p_1}$ specific heat of fluid [kcal/kg degC];

- $D_p$ diameter of solid particle [m] ;
- G. superficial mass velocity of fluid based on sectional area  $\lceil \frac{kg}{m^2h} \rceil$ ;
- $h_{r_s}$ heat-transfer coefficient of thermal radiation, solid surface to solid surface [kcal/m<sup>2</sup>h degC];
- heat-transfer coefhcient of thermal  $h_{r,s}$ radiation, void space to void space  $[kcal/m<sup>2</sup>h degC];$
- $h_w$ apparent wall-film coefficient of heat  $t_{w_1}, t_{w_2}$ , wall temperature  $[{}^{\circ}C]$ ; flowing  $[kcal/m^2h \text{ degC}]$ ;  $m$ ;
- $h_{\nu}^0$ apparent wall-film coefficient of heat transfer in packed bed with stagnant Greek symbols fluid  $[kcal/m<sup>2</sup>h degC]$ ;
- effective thermal conductivity in packed k<sub>e</sub> bed with fluid flowing  $[kcal/m h degC]$ ;
- effective thermal conductivity in packed  $k_s^0$ bed with stagnant fluid  $[kcal/m h degCl]$ ;
- $k_{e}^0$ effective thermal conductivity in packed bed with stagnant fluid in the vicinity of the wall [kcal/m h degC] ;
- thermal conductivity of fluid [kcal/m  $k_{f_{\star}}$ h degC];
- thermal conductivity of solid [kcal/m  $k_s$ h degC];
- h degC];  $\phi$ ,<br>thermal conductivity of marble plate  $\phi_w$ ,  $k_m$ [kcal/m h degC];  $\mu$ ,
- $L_{\star}$ thickness of marble plate [m];
- $p_{\rm F}$ emissivity of solid surface;
- pr, Prandtl number,  $= C_p \mu / k_f$ ;
- 4, heat flux  $[kcal/m<sup>2</sup>h]$ ;
- Rem, modified Reynolds number,  $= G D_p/\mu$ ;
- $t_{a_n}$ apparent wall temperature extrapolated from temperature distribution in packed bed to the wall surface  $[°C]$ ;
- $t'$ . temperature at  $D_p/2$  from the wall surface  $[^{\circ}C]$ :
- $t_m$ , mean temperature in packed bed  $[^{\circ}C]$ ;
- transfer in packed bed with fluid  $x$ , distance from surface of cooling jacket,
	- - (mass velocity in packed bed in the  $\alpha$ , direction of heat transfer)/(superficial mass velocity of fluid based on sectional area of empty tube in the direction of fluid flowing):
		- a-value in the vicinity of the wall;  $\alpha_{w}$ .
		- (effective length between two neighβ, bouring particles in the direction of heat flow)/ $D_p$ ;
		- void fraction; €,
		- void fraction in the vicinity of the wall;  $\epsilon_{w}$
		- (effective thickness of fluid film)/ $D_n$ ;
		- $\phi$ -value in the vicinity of the wall;
		- viscosity [kg/m h].

#### **INTRODUCTION**

IN **CONNECTION** with design calculations of catalytic reactors and heat exchangers of pebble heater type, it is very important to know the heattransfer properties of packed bed. In addition to a number of studies [l-6], Hatta and Maeda [7] analysed their experimental data of heat transfer in packed beds by means of their theoretical procedure including both the effective thermal conductivity  $k_e$  and the apparent wall-film coefficient of heat transfer  $h_w$ . Coberly and Marshall [l], Plautz and Johnstone [2], Yagi and Kunii [3], Campbell and Huntington [5], and then Maeda and Kawazoe [8] analysed their experimental data in almost the same way as Hatta and Maeda [7].

The effective thermal conductivity  $k_e^0$  of packed bed with stagnant fluid has been studied by a number of investigators, and there have been proposed many empirical and theoretical equations. Kunii and Smith [9] surveyed previous works, proposed a model for heat transfer in packed bed and gave a theoretical equation of stagnant conductivities, which takes account of thermal conductivity of both solid and fluid phases,

$$
\frac{k_e^0}{k_f} = \epsilon \left( 1 + \frac{h_{r_v} D_p}{k_f} \right) + \left[ (1 - \epsilon) \middle/ \left( \frac{1}{1/\Phi + h_{r_s} D_p / k_f} + \frac{2}{3} \frac{k_f}{k_s} \right) \right] \tag{1}
$$

where

$$
h_{r_v} = 0.1952 \cdot \left[ 1 \left/ \left( 1 + \frac{\epsilon}{2(1 - \epsilon)} - \frac{1 - p}{p} \right) \right] \times \frac{\left( \frac{t_m + 273}{100} \right)^3}{(2)}
$$
\n
$$
h_{r_s} = 0.1952 \frac{p}{2 - p} \left( \frac{t_m + 273}{100} \right)^3
$$
\n
$$
\phi = \phi_1 + (\phi_1 - \phi_2) \frac{\epsilon - 0.260}{0.216}
$$

Equation (1) can explain almost all experimental data of the effective thermal conductivities in packed beds with stagnant fluid, and can be used for their estimation.

On the other hand, several groups of investigators have measured the apparent wall coefficients of heat transfer  $h_w$  in packed beds through which gases were flowing. Coberly and Marshall [I], Campbell and Huntington [S], Plautz and Johnstone [2], Calderbank and Pogoroski [11] and Quinton and Storrow [12] correlated their experimental data by their empirical equations, Yagi and Wakao [13] by j-factors. Hanratty [14] and Yagi, Kunii and Shimomura [15] proposed semi-empirical equations, applying surface renewal theory and boundary-layer theory, respectively. Most of these equations are represented by the following form:

$$
\frac{h_w D_p}{k_f} = C \, Re_m^n \qquad (C, n : constant) \tag{3}
$$

However, this is not available for flow condition of small Reynolds numbers, because  $(k_w D_p/k_f)$ tends to zero when  $Re<sub>m</sub> = 0$ . From heat transfer measurements in annular packed beds at low *Re<sub>m</sub>*, Yagi and Kunii [16] ascertained that  $h_w$ does not reduce to zero when  $Re<sub>m</sub>$  tends to zero. They proposed the following theoretical equation for  $h_w$ .

$$
\frac{h_w D_p}{k_f} = \frac{h_w^0 D_p}{k_f} + \alpha_w Pr Re_m \tag{4}
$$

In equation (4)  $h_w^0$  represents the apparent wall-film coefficient of heat transfer in packed bed with stagnant fluid. Yagi and Kunii [17] have obtained the following equation based on their heat-transfer model of a packed bed:

$$
\frac{1}{h_w^0 D_p/k_f} = \frac{1}{k_w^0/k_f} - \frac{0.5}{k_e^0/k_f}
$$
 (5)

$$
\frac{k_w^0}{k_f} = \epsilon_w \left( 2 + \frac{h_{r_v} D_p}{k_f} \right) + \left[ (1 - \epsilon_w) / \left( \frac{1}{1/\phi_w + h_{rs} D_p/k_f} + \frac{1}{3} \cdot \frac{k_f}{k_s} \right) \right] (6)
$$

In these equations,  $k_w^0$  is twice of the effective thermal conductivity near wall  $k_{\epsilon w}^0$  (the distance of  $D_p/2$  from the wall). Though the first term of the right-hand side of equation (4),  $h_w^0 D_p/k_f$ , plays very important role at small *Rem,* there are a few experimental data of them. Some of them were obtained from extrapolation of previous data with flowing gases to  $Re_m = 0$  by Yagi and Kunii [16], and others were measured in annular packed beds by Yagi and Kunii [16] and Hill and Wilhelm [18]. Since equations (5) and (6) have not sufficient experimental backgrounds, the authors made a number of heat-transfer measurements in packed beds containing stagnant fluid where heat flowed completely in one direction in order to test the adequacy of the theoretical equations, namely equations (5) and (6).

# EXPERIMENT AND PROCEDURE

The experimental apparatus, similar to that used for the measurement of thermal conductivity of concrete solid, is shown schematically in Fig. 1. The upper and lower jackets have flat heat-transfer surfaces made of 15 mm thick copper plates which are held at constant temperatures by steam and water, respectively. An annular Dewar tube, made of acrylonitrile-resin, is used as the container of the packed bed to prevent radial heat loss. The packed bed, 200 mm in diameter and 50 mm in length, was formed upon a cylindrical marble plate which have the same dimensions as the packed bed. On the both flat surfaces of the marble plate are attached thermocouples to measure the surface temperatures. Thermal conductivity of marble plate was measured to be I.79

kcal/m h degC in the same apparatus by comparison with the thermal conductivities of copper and iron plates. The temperature distribution in packed bed was measured by ten thermocouples. The thermocouples used here were made of alumel-chrome1 and were 0.3 mm in diameter.

In the vicinity of the wall surface the orientation of particles and void fraction differ from those of the interior of the packed bed, resulting in a difference of the thermal conductivity near the wall from that of the inside. Thus the apparent wall-film coefficient of heat transfer  $h_w^0$  in packed beds is interpreted as a correction term when the thermal conductivity is assumed to be constant throughout the bed. Measurements of void distribution [19] showed that these wall effects were restricted to about half the diameter of a solid particle from the wall surface. Thus we may take two mean values of the effective thermal conductivities in packed bed, i.e.  $k_{e_m}^0$  near the wall surface (see Fig. 2) and  $k_e^0$  in the core of the bed.

In the present experiments, heat flowed completely in one direction, so that the temperature distribution in the packed bed was linear, as shown in Fig. 2. The heat flux is expressed as:



2. Steam jacket

- 
- 3. Cooling water jacket 8. Fluid inlet<br>
4. Annular Dewar tube 9. Rubber gasket 4. Annular Dewar tube
- j. Marble plate
- 
- 
- 



FIG. 2. Schematic diagram of temperature distribution in packed bed.

$$
q = k_e^0 \frac{dt}{dx} = k_{e_w}^0 \frac{t' - t_{w_1}}{D_p/2} = h_w^0 (t_{a_p} - t_{w_1})
$$
  
=  $k_m \frac{t_{w_1} - t_{w_2}}{L}$  (7)

Temperature gradients in packed bed  $dt/dx$ were obtained by averaging of temperature distribution by delta-sigma method. From equation (7) the following expression is derived,

$$
\frac{1}{k_e^0} + \frac{1}{h_w^0 \ D_p/2} = \frac{1}{k_{e_m}^0} \tag{8}
$$

Comparing equation (5) with equation (8), it is found that  $k_{e_{w}}^{0} = 0.5 k_{w}^{0}$ .

#### **RESULTS**

Measurements were made for each of twelve kinds of solid particles with five fluids, i.e. water, air, carbon dioxide, helium and hydrogen. Experimental conditions are shown in Table 1. Reproductivity of results was examined by repacking of solid particles. Comparison of experimental results with the values of calculated by the authors' theoretical equations are shown in Table 2, Figs. 4 and 5. In Fig. 4,  $k_{ew}^0/k_f$  are plotted against  $k_s/k_f$ , in which the solid line illustrates equation (6), with the assumption that  $k_{e_m}^0 = 0.5 \; k_w^0$  and  $\epsilon_w = 0.40$ , with no radiation contribution. Figure 5 shows  $h_w^0 D_p/k_f$  vs.  $k_s/k_f$ )

*Table 1. Experimental conditions* 

Solid	Glass spheres $(D_p = 1.15, 2.26,$ $3.69, 6.38, 8.70$ and $12.1$ mm)
	Steel balls $(D_n = 3.09, 6.32$ and $10-9$
	Raschig rings ( $D_n = 5.12$ and 8.18)
	Cement clinckers ( $D_p = 7.31$ )
	Nickel catalyst pellets (cylinder,
	$D_n = 4.75$
	River sands (the Akikawa river,
	$D_n = 2.45$
Fluid	water, air, carbon dioxide, helium and hydrogen
void fraction	$0.338 \sim 0.603$
bed temperature	$30 \sim 90^{\circ}$ C, $t_m = 50^{\circ}$ C

with a solid line calculated from equation (8),  $k_{e_m}^0$  and  $k_e^0$  from equations (6) and (1), where  $\epsilon_w = 0.40$  and  $\epsilon = 0.34$ , respectively.

## *COMPARISON* OF CALCULATED AND OBSERVED VALUES

Equation (1) was applied for calculation of stagnant thermal conductivities  $k_e^0$  in packed beds and equation (6) with  $k_{ew}^0 = 0.5 k_w^0$  for the effective thermal conductivities in the vicinity of the wall surface with stagnant fluid  $k_{\epsilon}^0$ . The values of  $k_s$ ,  $k_f$  and  $p$  were taken from physical tables [20]. The unknown term  $\epsilon_w$ , average void fraction in the vicinity of the wall surface, was estimated as the following way. In the present measurements, as solid particles were packed upon a horizontal plate, it was observed that the arrangement of particles near wall was almost in the closest packing. Thus  $\epsilon_w$  was determined the volume average void fraction from the wall surface to  $D_p/2$  for the closest packing of spherical particles on a horizontal plate, i.e.  $\epsilon_w = 1 - \pi/(3\sqrt{3}) = 0.40$ . The quantities  $\phi_1$ ,  $\phi_2$  and  $\phi_w$  can be read from Fig. 3, which was taken from reference [4]. Thus calculated values of  $k_e^0$  and  $k_{e}^0$  were compared with the observed ones, Figs. 6 and 7, where the average deviations are smaller than 15 and 25 per cent, respectively. Then it can be reasonably concluded that equation (6) can be used to estimate  $k_{em}^0$ , so that  $h_m^0$  from equations (1), (6) and (8). In case of ordinary packed beds, the arrangement of particles in the vicinity of the wall may be different from the present case, i.e. the packed









FIG. 5.  $h_w^0D_p/k_f$  vs.  $k_s/k_f$ 



754

# KUNIHIKO OFUCHI and DAIZO KUNII





HARACTERISTICS OF PACKED BEDS WITH STAGNANT FLUIDS



FIG. 6. Comparison of experimental and calculated values of  $k_e^0/k_f$ .



values of  $k_{\epsilon_m}^0/k_f$ .

bed on a horizontal plate. However, this effect can be attributed to  $\epsilon_w$  alone, which may be estimated as the function of the ratio of the diameter of solid particles to that of the tube and of the mean void fraction in the packed bed. Therefore, equation (6) can be used for estimating  $k_{e_{w}}^{0}$  of the packed bed in a tube. 209 (1954).

#### **CONCLUSIONS**

(1) Heat-transfer measurements were made in packed beds with stagnant fluid. Finite values of the apparent wall-film coefficients of heat transfer were observed. Thus at small *Rem* equations (3) cannot be used for estimating  $h_w$ .

(2) Many experimental values of  $k_e^0$ ,  $k_{e_w}^0$  and  $h_w^0$  were obtained under various conditions, from which it was found that the theoretical equations proposed by the authors [17], equations (5) and (6), can give good estimation of  $h^0_{\omega}$ .

#### **REFERENCES**

- **I. C. A. COBERLY and W. R. MARSHALL JR., Tempera** ture gradient in gas streams flowing through fixed granular beds, Chem. Engng Progr. 47, 141 (1951).
	- **2.**  D. A. PLAUTZ and H. F. JOHNSTONE, Heat and mass transfer in packed beds, J. *Amer. Inst. Chem.* **Engrs**  1, 193 (1951).
	- **3.**  S. YAGI and D. KIJNII, Studies on effective thermal conductivities in packed beds, *Kagaku-Kogaku, Chem. Engng Japan* **18,** *576 (1954); J. Amer. Inst. Chem. Engrs 3, 373 (1957).*
	- *S.* YAGI, D. KUNII and N. WAKAO, Radially effective thermal conductivities in packed **beds,**  *International Development in Heat Transfer,* Part IV, p. 742 (1962).
	- J. M. CAMPBELL and R. L. HUNTINGTON, Heat transfer and pressure drop in spherical and cylindrical solids, *Petrol. Refiner 31, 123 (1952).*
	- M. KIMURA, Effective thermal conductivities of packed beds, *Kagaku-Kogaku, Chem. Engng Japan 21, 472 (1957).*
	- S. HATTA and S. MAEDA, Heat transfer in beds of granular catalyst, I and II, *Kagaku-Kikai, Chem. Engng Japan 12, 56 (1948); 13,79 (1949).*
	- *S.* MAEDA and K. KAWAZOE, Heat transfer in beds of granular catalyst, *Kagaku-Kogaku, Chem. Engng Japan 15, 5, 9, 312 (1951).*
	- 9. D. KUNII and J. M. SMITH, Heat transfer characteristics of porous rocks, *J. Amer. Inst.* **Chem.** *Engrs* **6, 71 (1960).**
	- 10. **G.** DAMKBLER, Einfliisse der StrGmung, Diffusion und des Wärme Überganges auf die Leistung von ReaktionsGfen, *Chem. lg. 3, 441 (1932); Z. Elektr. Chem. 42, 846 (1936).*
	- 11. P. H. CALDERBANK and J. A. POGOROSKI, Heat transfer in packed beds, *Trans. Chem. Engrs, Lond.* 35, 195 (1957).
	- J. H. *QUINTON* and J. A. STORROW, Heat transfer to air flowing through packed tubes, *Chem. Engng Sci. 5, 245 (1956).*
	- *S.* **YAGI** and N. WAKAO, Heat and mass transfer from wall to fluid in packed beds, *J. Amer. Inst. Chem. Engrs 5, 79 (1957).*
	- **14.** T. J. HANRATTY, Nature of wall heat transfer coefficient in packed beds, *Chem. Engng Sci.* 5, 209 (1954).
- 15. S. **YAGI,** D. KUNII and Y. **SHIMOMURA,** Studies on heat transfer in packed beds with fluid flow, *Kagaku-Kogaku, Chem. Engng Japan 21,342 (1957).*
- *16. S. YAGI* and D. KUNII, Studies on heat transfer near wall in packed beds, *J. Amer. Inst. Chem. Engrs 6, 97 (1960).*
- 17. S. YAGI and D. KUNII, Studies on heat transfer in packed beds, *International Development in Heat Transfer,* Part IV, p. 750 (1962).
- 18. F. B. HILL and R. H. WILHELM, Radiative and conductive heat transfer in quiescent gas-solid bed of particles: Theory and experiment, *J. Amer. Inst. Chem. Engrs 5, 489 (1959).*
- *19.* **M.** KIMURA and T. KANEDA, Distribution of void in packed tube, *Kagaku-Kogaku, Chem. Engng Japan 19, 397 (1955).*
- *20.* J. H. PERRY, *Chemical Engineers' Handbook* (3th ed.), p. *456.* McGraw-Hill, New York (1950).

Résumé—Les distributions de température ont été mesurées en régime permanent dans des lits fixes de particules solides avec des fluides au repos, à partir desquelles les conductivités thermiques effectives et les coefficients apparents de transfert de chaleur par film pariétal ont été obtenus à la fois. Les mesures ont été faites pour chacune de douze espèces de particules avec cinq fluides, c'est-à-dire l'eau, l'air, le gaz carbonique, l'hélium et l'hydrogène.

Les données expérimentales des conductivités thermiques effectives montraient un bon accord avec les équations théoriques proposées auparavant par les auteurs.

L'équation théorique de film pariétal a été modifiée ici et appliquée à l'analyse des données expéri**mentales actuelles.** 

Zusammenfassung-Im stationären Zustand wurden die Temperaturverteilungen in Schüttungen fester Teilchen mit stillstehenden Flüssigkeiten gemessen, wodurch man sowohl das tatsächliche Wärmeleitvermögen wie auch die scheinbaren Wärmeübergangszahlen des Wandfilmes erhält. Für jede von zwiilf Arten von Teilchen wurden mit fiinf Fliissigkeiten, d.h. Wasser, Luft, Kohlendioxyd, Helium und Wasserstoff Messungen gemacht.

Die Versuchsergebnisse des tatsächlichen Wärmeleitvermögens zeigten gute Übereinstimmung mit den von den Autoren vorher vorgeschlagenen Gleichungen.

Die theoretische Gleichung der Autoren fiir den scheinbaren Wandfilmkoeffizienten wurde hier modifiziert und bei der Analyse der vorliegenden Versuchsergebnisse angewendet.

Аннотация-Измерялось распределение температур в стационарном состоянии плотных слоев твердых частиц в застойных течениях, что позволило получить как эффективные коэффициенты теплопроводности, так и отдельно коэффициенты для пленок на стенке. Измерения проводились для частиц двенадцати видов в пяти жидкостях-в воде, воздухе, двуокиси углерода, гелии и водороде.

 $\partial \tilde{\phi}$ фективные коэффициенты теплопроводности, полученные экспериментально, хорошо согласуются с теоретическими уравнениями, предложенными авторами ранее.

В данной статье теоретическое уравнение авторов для истинного пленочного коэффициента на стенке изменено и применяется для анализа представленных экспери-**MeHTaJIbHhIX AaHHbIX.**